Finite Math - Spring 2019

Lecture Notes - 2/21/2019

Homework

- Section 3.1 - 79, 81
- Section 3.2 - 37, 39, 41, 43, 65, 66, 67, 68, 69, 70

Section 3.1 - Simple Interest

Average Daily Balance. A common method for calculating interest on a credit card is to use the *average daily balance method*. As the name suggests, the average daily balance is computed, then the interest is computed on that.

Example 1. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30*-day billing cycle is* \$523.18 *and purchases of* \$147.98 *and* \$36.27 *are posted on days* 12 *and* 25*, respectively, and a payment of* \$200 *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

Example 2. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* $\$696.21$ *and purchases of* $\$25.59$, $\$19.95$, *and* $\$97.26$ *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* $\$140$ *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

Solution.

Section 3.2 - Compound and Continuous Compound Interest

Compound Interest. In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example 3. *Suppose* $\$5,000$ *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

Solution.

If we generalize this process, we end up with the following result

Definition 1 (Compound Interest)**.**

The variables in this equation are

- $A =$ *future value after* n *compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $n =$ *total number of compounding periods*

Alternately, one can reinterpret this formula as a function of time as

where A, P, r, and m have the same meanings as above and t is the time in years.

Example 4. *If* $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution.

Example 5. *If* $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after* 5 *years? How much interest is accrued in each case? Round answers to the nearest cent.*

Solution.

Continuous Compound Interest. Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

Definition 2 (Continuous Compound Interest)**.** *Principal* P *invested at an annual nominal rate* r *will have future value*

after time t *(in years).*

Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.

Example 6. *If* $\$1,000$ *is invested at* 6% *interest compounded continuously, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution.

Example 7. *If* $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) continuously, what is the amount after* 5 *years? Round answers to the nearest cent. (Assume* 365 *days in a year.)*

Solution.